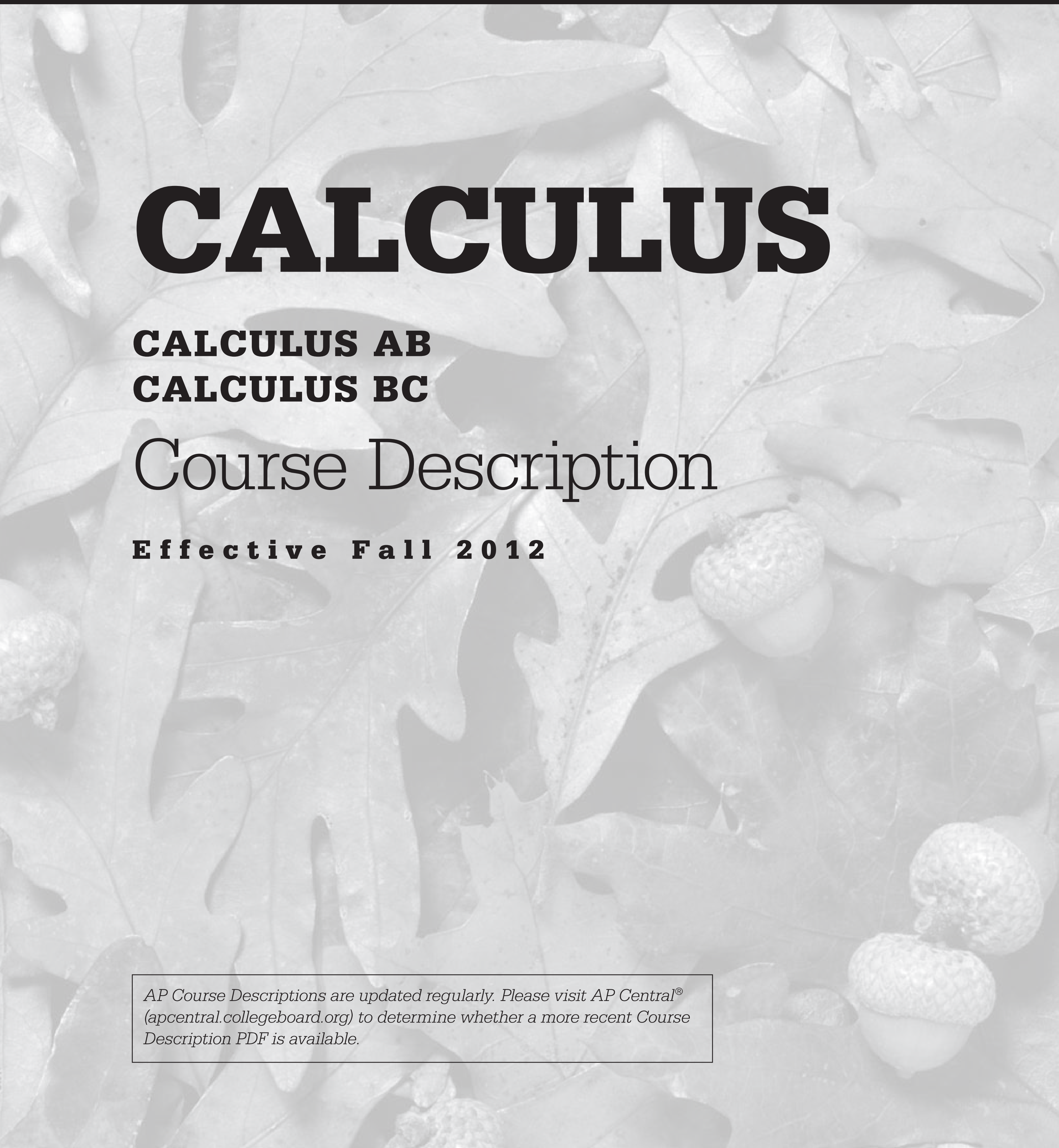

CollegeBoard AP

CALCULUS

CALCULUS AB
CALCULUS BC

Course Description

Effective Fall 2012

AP Course Descriptions are updated regularly. Please visit AP Central® (apcentral.collegeboard.org) to determine whether a more recent Course Description PDF is available.

The College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 5,900 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

For further information, visit **www.collegeboard.org**.

AP Equity and Access Policy

The College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underserved. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. The College Board also believes that all students should have access to academically challenging course work before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

AP Course Descriptions

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Contents

About the AP® Program

AP® enables students to pursue college-level studies while still in high school. Through more than 30 courses, each culminating in a rigorous exam, AP provides willing and academically prepared students with the opportunity to earn college credit, advanced placement, or both. Taking AP courses also demonstrates to college admission officers that students have sought out the most rigorous course work available to them.

Each AP course is modeled upon a comparable college course, and college and university faculty play a vital role in ensuring that AP courses align with college-level standards. Talented and dedicated AP teachers help AP students in classrooms around the world develop and apply the content knowledge and skills they will need in college.

Each AP course concludes with a college-level assessment developed and scored by college and university faculty as well as experienced AP teachers. AP Exams are an essential part of the AP experience, enabling students to demonstrate their mastery of college-level course work. More than 90 percent of four-year colleges and universities in the United States grant students credit, placement, or both on the basis of successful AP Exam scores. Universities in more than 60 countries recognize AP Exam scores in the admission process and/or award credit and placement for qualifying scores. Visit www.collegeboard.org/ap/creditpolicy to view AP credit and placement policies at more than 1,000 colleges and universities.

Performing well on an AP Exam means more than just the successful completion of a course; it is a pathway to success in college. Research consistently shows that students who score a 3 or higher on AP Exams typically experience greater academic success in college and are more likely to graduate on time than otherwise comparable non-AP peers. Additional AP studies are available at www.collegeboard.org/apresearchsummaries.

Offering AP Courses and Enrolling Students

This course description details the essential information required to understand the objectives and expectations of an AP course. The AP Program unequivocally supports the principle that each school develops and implements its own curriculum that will enable students to develop the content knowledge and skills described here.

Schools wishing to offer AP courses must participate in the AP Course Audit, a process through which AP teachers' syllabi are reviewed by college faculty. The AP Course Audit was created at the request of College Board members who sought a means for the College Board to provide teachers and administrators with clear guidelines on curricular and resource requirements for AP courses and to help colleges and universities validate courses marked "AP" on students' transcripts. This process ensures that AP teachers' syllabi meet or exceed the curricular and resource expectations that college and secondary school faculty have established for college-level courses. For more information on the AP Course Audit, visit www.collegeboard.org/apcourseaudit.

How AP Courses and Exams Are Developed

AP courses and exams are designed by committees of college faculty and expert AP teachers who ensure that each AP subject reflects and assesses college-level expectations. AP Development Committees define the scope and expectations of the course, articulating through a curriculum framework what students should know and be able to do upon completion of the AP course. Their work is informed by data collected from a range of colleges and universities to ensure that AP coursework reflects current scholarship and advances in the discipline. To find a list of each subject's current AP Development Committee members, please visit apcentral.collegeboard.org/developmentcommittees.

The AP Development Committees are also responsible for drawing clear and well-articulated connections between the AP course and AP Exam — work that includes designing and approving exam specifications and exam questions. The AP Exam development process is a multi-year endeavor; all AP Exams undergo extensive review, revision, piloting, and analysis to ensure that questions are high quality and fair, and that there is an appropriate spread of difficulty across the questions.

Throughout AP course and exam development, the College Board gathers feedback from various stakeholders in both secondary schools and higher education institutions. This feedback is carefully considered to ensure that AP courses and exams are able to provide students with a college-level learning experience and the opportunity to demonstrate their qualifications for advanced placement upon college entrance.

How AP Exams Are Scored

The exam scoring process, like the course and exam development process, relies on the expertise of both AP teachers and college faculty. While multiple-choice questions are scored by machine, the free-response questions are scored by thousands of college faculty and expert AP teachers at the annual AP Reading. AP Exam Readers are thoroughly trained, and their work is monitored throughout the Reading for fairness and consistency. In each subject, a highly respected college faculty member fills the role of Chief Reader, who, with the help of AP Readers in leadership positions, maintains the accuracy of the scoring standards. Scores on the free-response questions are weighted and combined with the weighted results of the computer-scored multiple-choice questions. These composite, weighted raw scores are converted into the reported AP Exam scores of 5, 4, 3, 2, and 1.

The score-setting process is both precise and labor intensive, involving numerous psychometric analyses of the results of a specific AP Exam in a specific year and of the particular group of students who took that exam. Additionally, to ensure alignment with college-level standards, part of the score-setting process involves comparing the performance of AP students with the performance of students enrolled in comparable courses in colleges throughout the United States. In general, the AP composite score points are set so that the lowest raw score needed to earn an AP Exam score of 5 is equivalent to the average score among college students earning grades of A in the college course. Similarly, AP Exam scores of 4 are equivalent to college grades of A–, B+, and B. AP Exam scores of 3 are equivalent to college grades of B–, C+, and C.

AP Score	Qualification
5	Extremely well qualified
4	Well qualified
3	Qualified
2	Possibly qualified
1	No recommendation

Additional Resources

Visit apcentral.collegeboard.org for more information about the AP Program.

AP Calculus

INTRODUCTION

AP courses in calculus consist of a full high school academic year of work and are comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the exam for each course. The two courses and the two corresponding exams are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. This curriculum should include all the prerequisites for a year's course in calculus listed on page 6. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still teach the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB Exam, most of the year must be devoted to the topics in differential and integral calculus described on pages 6 to 9. These topics are the focus of the AP Exam questions.

Calculus BC can be offered by schools where students are able to complete all the prerequisites listed on page 6 before taking the course. Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics taught in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics. The topics for Calculus BC are described on pages 9 to 12. A Calculus AB subscore is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Exam or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide statements regarding their AP policies in their catalogs and on their websites.

Secondary schools have a choice of several possible actions regarding AP Calculus. The option that is most appropriate for a particular school depends on local conditions and resources: school size, curriculum, the preparation of teachers, and the interest of students, teachers, and administrators.

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses that are the equivalent of four full years of high school mathematics before attempting calculus. These courses should include the study of algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies — including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study — it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

The AP Calculus Development Committee recommends that calculus should be taught as a college-level course. With a solid foundation in courses taken before AP, students will be prepared to handle the rigor of a course at this level. Students who take an AP Calculus course should do so with the intention of placing out of a comparable college calculus course. This may be done through the AP Exam, a college placement exam, or any other method employed by the college.

THE COURSES

Philosophy

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central [apcentral.collegeboard.org] for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f and f'.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Derivative rules for sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity).

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Numerical approximations to definite integrals. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.

- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

* **Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f and f'.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.

\+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.

- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

\+ Numerical solution of differential equations using Euler's method.

\+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Derivative rules for sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

+ Derivatives of parametric, polar, and vector functions.

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity).

* **Applications of integrals.** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions.

+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
+ Improper integrals (as limits of definite integrals).

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

+ Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

*IV. Polynomial Approximations and Series

* **Concept of series.** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

* **Series of constants**

+ Motivating examples, including decimal expansion.
+ Geometric series with applications.
+ The harmonic series.
+ Alternating series with error bound.
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
+ The ratio test for convergence and divergence.
+ Comparing series to test for convergence or divergence.

* **Taylor series**

+ Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).
+ Maclaurin series and the general Taylor series centered at $x = a$.
+ Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
+ Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
+ Functions defined by power series.
+ Radius and interval of convergence of power series.
+ Lagrange error bound for Taylor polynomials.

USE OF GRAPHING CALCULATORS

Professional mathematics organizations such as the National Council of Teachers of Mathematics, the Mathematical Association of America, and the Mathematical Sciences Education Board of the National Academy of Sciences have strongly endorsed the use of calculators in mathematics instruction and testing. The use of a graphing calculator in AP Calculus is considered an integral part of the course.

As stated in the Goals on page 6, students should learn how to use technology to help solve problems, experiment, interpret results, and support conclusions. Significant among these uses are discovery (experiment) and reflection (interpret results/support conclusions). Examples include zooming to reveal local linearity, constructing a table of values to conjecture a limit, developing a visual representation of Riemann sums approaching a definite integral, graphing Taylor polynomials to understand intervals of convergence for Taylor series, or using the calculator to draw a slope field and investigate how the choice of initial condition affects the solution to a differential equation. Many of the teacher resources on AP Central provide support for the use of graphing calculators in classroom instruction.

Graphing calculators are a valuable investigative tool for calculus explorations, and the fruits of these student investigations are assessed on the AP Calculus Exams. However, there is not enough time under the constraints of the AP Exam to allow students to fully employ exploration techniques with graphing calculators. Thus, graphing calculators are used on the exam directly toward solving problems.

The sections of the AP Calculus Exams are described on page 16. Both Section I (multiple choice) and Section II (free response) include problems that require the use of a graphing calculator. Beginning with the May 2011 exams, the format of the free-response sections of the AP Calculus Exams was modified so that Part A (graphing calculator required) consists of two problems and Part B (no calculator is allowed) consists of four problems.

The change in format should not impact classroom instruction. In particular, students should use graphing calculators on a regular basis so that they become adept in their use. Students should also have experience with the basic paper-and-pencil techniques of calculus and be able to apply them when technological tools are unavailable or inappropriate. The Development Committee believes that the change in exam format will help the AP Calculus Exams more accurately represent the broad range of calculus topics and concepts that need to be assessed.

The AP Calculus Development Committee understands that new calculators and computers capable of enhancing the teaching and learning of calculus continue to be developed. There are two main concerns that the committee considers when deciding what level of technology should be required for the exams: equity issues and teacher development.

Over time, the range of capabilities of graphing calculators has increased significantly. Some calculators are much more powerful than first-generation graphing calculators and may include symbolic algebra features. Other graphing calculators are, by design, intended for students studying mathematics at lower levels than calculus. The committee can develop exams that are appropriate for any given level of technology, but it cannot develop exams that are fair to all students if the spread in the capabilities of the technology is too wide. Therefore, the committee has found it necessary to make certain requirements of the technology that will help ensure that all students have sufficient computational tools for the AP Calculus Exams. Exam restrictions should not be interpreted as restrictions on classroom activities. The committee will continue to monitor the developments of technology and will reassess the testing policy regularly.

Graphing Calculator Capabilities for the Exams

The committee develops exams based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

1. Plot the graph of a function within an arbitrary viewing window
2. Find the zeros of functions (solve equations numerically)
3. Numerically calculate the derivative of a function
4. Numerically calculate the value of a definite integral

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any exam question that requires the use of a calculator. Care is taken to ensure that the exam questions do not favor students who use graphing calculators with more extensive built-in features.

Students are expected to bring a graphing calculator with the capabilities listed above to the exams. AP teachers should check their own students' calculators to ensure that the required conditions are met. Students and teachers should keep their calculators updated with the latest available operating system. Information is available on calculator company websites. A list of acceptable calculators can be found at AP Central. Teachers must contact the AP Program (609-771-7300) before April 1 of the testing year to inquire whether a student can use a calculator that is not on the list. If the calculator is approved, written permission will be given.

Technology Restrictions on the Exams

Computers, electronic writing pads, pocket organizers, nongraphing scientific calculators, and calculator models with any of the following are <u>not permitted</u> for use on the AP Calculus Exams: QWERTY keypads as part of hardware or software, pen-input/stylus/touch-screen capability, wireless or Bluetooth® capability, paper tapes, "talking" or noise-making capability, need for an electrical outlet, ability to access the Internet, cell phone capability or audio/video recording capability, digital audio/video players, or camera or scanning capability. In addition, the use of hardware peripherals with an approved graphing calculator is not permitted.

Test administrators are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. The student should be thoroughly familiar with the operation of the calculator he or she plans to use. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the approved list.

Calculator memories will not be cleared. Students are allowed to bring calculators containing whatever programs they want. They are expected to bring calculators that are set to radian mode.

Students must not use calculator memories to take test materials out of the room. Students should be warned that their scores will be invalidated if they attempt to remove test materials by any method.

Showing Work on the Free-Response Sections

An important goal of the free-response sections of the AP Calculus Exams is to provide students with an opportunity to communicate their knowledge of correct reasoning and methods. Students are required to show their work so that AP Exam Readers can assess the students' methods and answers. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly. Answers without supporting work will usually not receive credit. Students should use complete sentences in responses that include explanations or justifications.

For results obtained using one of the four required calculator capabilities listed on page 14, students are required to write the mathematical setup that leads to the solution along with the result produced by the calculator. These setups include the equation being solved, the derivative being evaluated, or the definite integral being evaluated. For example, if a problem involves finding the area of a region, and the area is appropriately computed with a definite integral, students are expected to show the definite integral — written in standard mathematical notation — and the answer. In general, in a calculator-active problem that requires the value of a definite integral, students may use a calculator to determine the value; they do not need to compute an antiderivative as an intermediate step. Similarly, if a calculator-active problem requires the value of a derivative of a given function at a specific point, students may use a calculator to determine the value; they do not need to state the symbolic derivative expression. For solutions obtained using a calculator capability other than one of the four listed on page 14, students must show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if students are asked to find a relative minimum value of a function, they are expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a calculator application that finds minimum values.

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a substitute for a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution. Therefore, when students are asked to justify or explain an answer, the justification must include mathematical, noncalculator reasons, not merely calculator results. Also, within solutions and justifications of answers, any functions, graphs, tables, or other objects that are used must be clearly labeled.

As on previous AP Calculus Exams, if a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated in the problem. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology. For example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

Sign charts by themselves are not accepted as a sufficient response when a free-response problem requires a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. Rather, the justification must include a clear explanation about how the behavior of the derivative and/or second derivative of the function indicates the particular extremum. For more detailed information on this policy, read the article "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema," which is available on the Calculus AB and Calculus BC Exam Pages at AP Central.

For more information on the instructions for the free-response sections, read the "2011 FRQ Instruction Commentary," which is available on the Calculus AB and Calculus BC Exam Pages at AP Central.

THE EXAMS

The Calculus AB and BC Exams seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each exam consists of two sections, as described below.

Section I: a multiple-choice section testing proficiency in a wide variety of topics

Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning

The time allotted for each AP Calculus Exam is 3 hours and 15 minutes. The multiple-choice section of each exam consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

Multiple-choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, students are encouraged to answer all multiple-choice questions. On any questions students do not know the answer to, students should eliminate as many choices as they can, and then select the best answer among the remaining choices.

The free-response section of each exam has two parts: one part for which graphing calculators are required, and a second part for which calculators are prohibited. The AP Exams are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater flexibility in the types of problems that can be given while ensuring fairness to all students taking the exam, regardless of the graphing calculator used.

The free-response section of each exam consists of six problems in 90 minutes. Part A of the free-response section (two problems in 30 minutes) requires the use of a graphing calculator. Part B of the free-response section (four problems in 60 minutes) does not allow the use of a calculator. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

In determining the score for each exam, the scores for Section I and Section II are given equal weight. Since the exams are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Calculus AB Subscore for the Calculus BC Exam

A Calculus AB subscore is reported based on performance on the portion of the exam devoted to Calculus AB topics (approximately 60 percent of the exam). The Calculus AB subscore is designed to give colleges and universities more information about the student. Although each college and university sets its own policy for awarding credit and/or placement for AP Exam scores, it is recommended that institutions apply the same policy to the Calculus AB subscore that they apply to the Calculus AB score. Use of the subscore in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 27.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

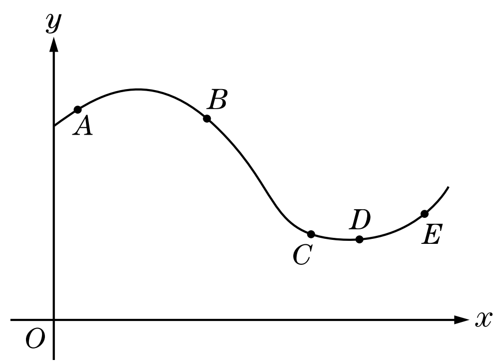
(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. What is $\lim\limits_{h \to 0} \dfrac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

   (A) 1
   (B) $\dfrac{\sqrt{2}}{2}$
   (C) 0
   (D) -1
   (E) The limit does not exist.

2. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?



   (A) A
   (B) B
   (C) C
   (D) D
   (E) E

3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

   (A) $-\dfrac{3}{2}$
   (B) -1
   (C) $-\dfrac{5}{14}$
   (D) $-\dfrac{3}{14}$
   (E) 0

4. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

   (A) $\pi \int_0^1 \left(\left(3 - 2x^2\right)^2 - (3 - 2x)^2 \right) dx$

   (B) $\pi \int_0^1 \left((3 - 2x)^2 - \left(3 - 2x^2\right)^2 \right) dx$

   (C) $\pi \int_0^1 \left(4x^2 - 4x^4\right) dx$

   (D) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2 \right) dy$

   (E) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2 \right) dy$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

   (A) The function has no relative extremum.
   (B) The graph of the function has one point of inflection and the function has two relative extrema.
   (C) The graph of the function has two points of inflection and the function has one relative extremum.
   (D) The graph of the function has two points of inflection and the function has two relative extrema.
   (E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

   (A) $-2 \cos 3$
   (B) $-2 \sin 3 \cos 3$
   (C) $6 \cos 3$
   (D) $2 \sin 3 \cos 3$
   (E) $6 \sin 3 \cos 3$

7. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

   (A) $y = 2x$ for $x > 0$

   (B) $y = 2x - 6$ for $x \ne 3$

   (C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

   (D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

   (E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

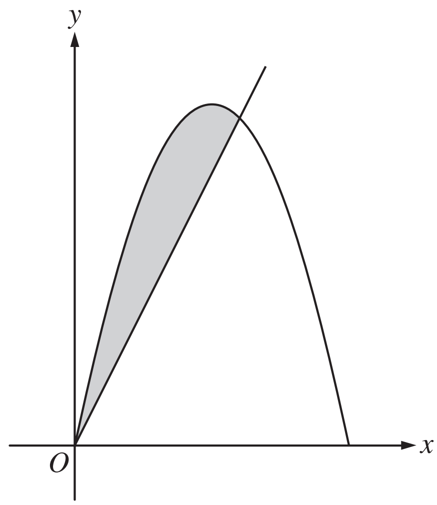
(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

10. $\int (x-1)\sqrt{x}\, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
(B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$
(C) $\frac{1}{2}x^2 - x + C$
(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$
(E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$

11. What is $\lim\limits_{x\to\infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) The limit does not exist.



12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

13. If $y = 5 + \int_2^{2x} e^{-t^2}\,dt$, which of the following is true?

(A) $\frac{dy}{dx} = e^{-x^2}$ and $y(0) = 5$

(B) $\frac{dy}{dx} = e^{-x^2}$ and $y(1) = 5$

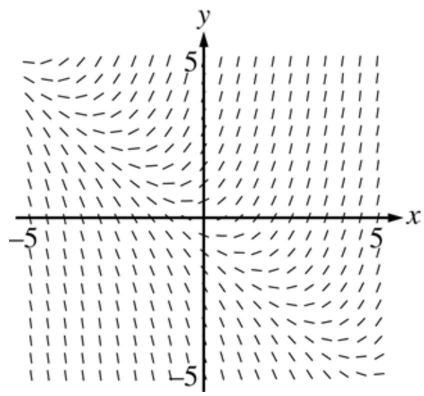
(C) $\frac{dy}{dx} = e^{-4x^2}$ and $y(1) = 5$

(D) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(0) = 5$

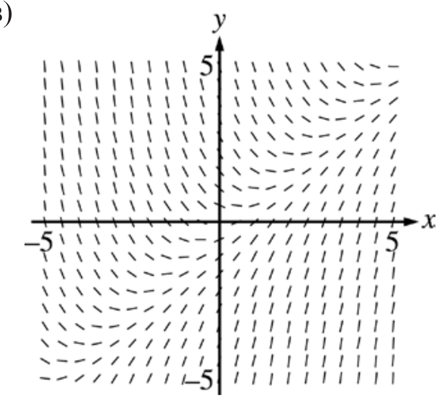
(E) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(1) = 5$

14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

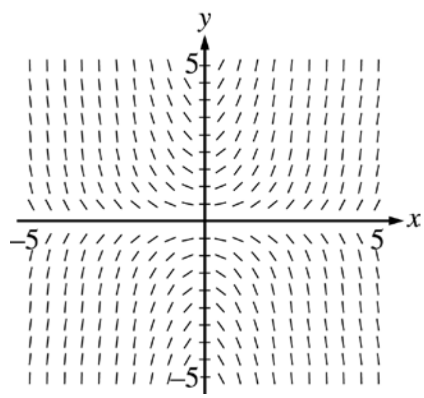
(A)



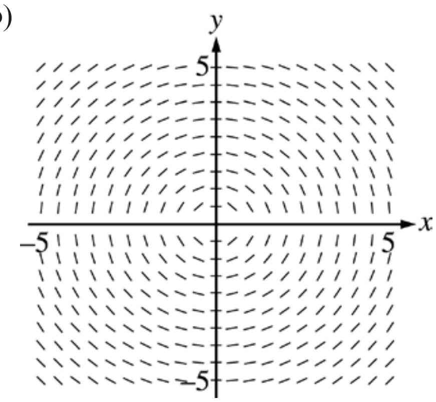
(B)



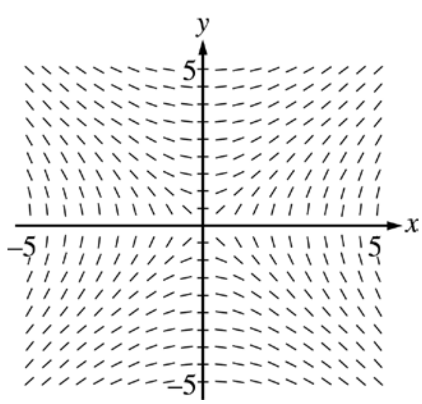
(C)



(D)



(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. Following are the directions for Section I, Part B, and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

16. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

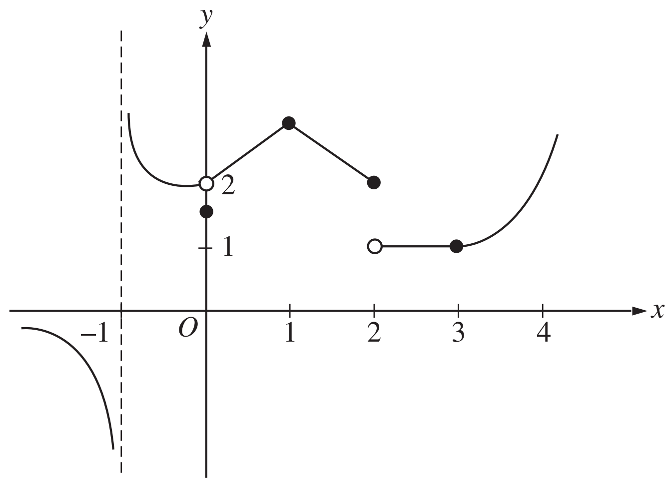
(A) $2\pi\int_0^1 r\,f(r)\,dr$

(B) $2\pi\int_0^1 r(1+f(r))\,dr$

(C) $2\pi\int_0^2 r(1+f(r))\,dr$

(D) $2\pi\int_1^2 r\,f(r)\,dr$

(E) $2\pi\int_1^2 r(1+f(r))\,dr$



17. The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) -1
(B) 0
(C) 1
(D) 2
(E) 3

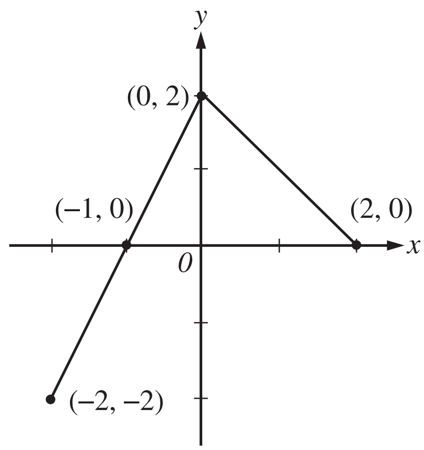
x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

18. Let f be a function such that $f''(x) < 0$ for all x in the closed interval [1, 2]. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four



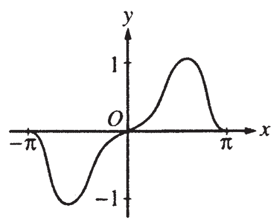
Graph of f

20. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

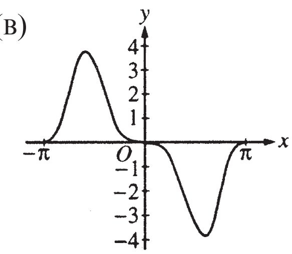
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

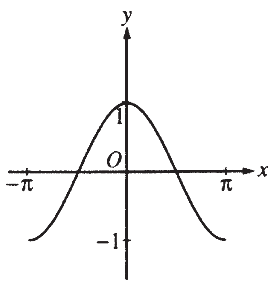
(A)



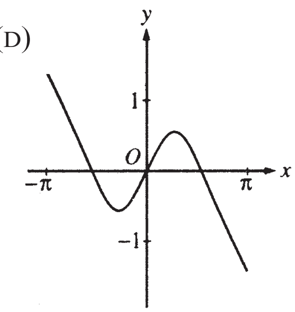
(B)



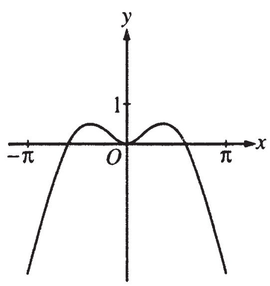
(C)



(D)



(E)



22. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using the local linear approximation for f at $x = 5$?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(E) 1.157

Answers to Calculus AB Multiple-Choice Questions

Part A		*Part B*	
1.	A	15.*	D
2.	B	16.	D
3.	C	17.	B
4.	A	18.	D
5.	C	19.*	D
6.	B	20.	B
†7.	C	21.	E
8.	C	22.	A
9.	E	23.*	D
10.	D	24.*	E
11.	B		
12.	B		
13.	E		
14.	E		

*Indicates a graphing calculator-active question.
†For resources on differential equations, see the Course Home Pages for Calculus AB and Calculus BC at AP Central.

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 39.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

(A) $2x - 3y = 0$
(B) $4x - 5y = 2$
(C) $4x - y = 10$
(D) $5x - 4y = 7$
(E) $5x - y = 13$

2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x + y}{y^2}$

(B) $-\frac{3x + y}{x + y}$

(C) $\frac{1 - 3x - y}{x + y}$

(D) $-\frac{3x}{1 + y}$

(E) $-\frac{3x}{x + y}$

x	$g'(x)$
-1.0	2
-0.5	4
0.0	3
0.5	1
1.0	0
1.5	-3
2.0	-6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) -6.5
(B) -1.5
(C) 1.5
(D) 2.5
(E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

(A) All x except $x = 0$
(B) $|x| = 3$
(C) $-3 \le x \le 3$
(D) $|x| > 3$
(E) The series diverges for all x.

5. If $\dfrac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\dfrac{d}{dx} f(h(x)) =$

(A) $g\left(x^2\right)$

(B) $2xg(x)$

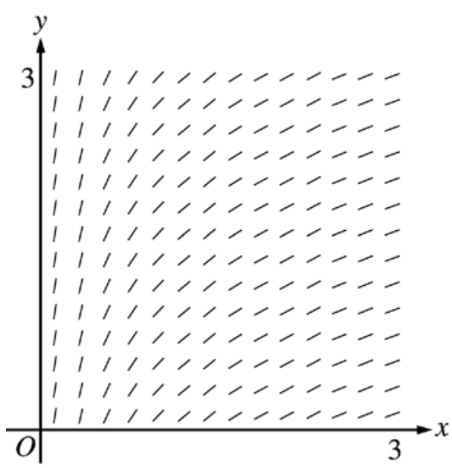
(C) $g'(x)$

(D) $2xg\left(x^2\right)$

(E) $x^2 g\left(x^2\right)$

6. If F' is a continuous function for all real x, then $\lim\limits_{h \to 0} \dfrac{1}{h} \displaystyle\int_a^{a+h} F'(x)\, dx$ is

(A) 0
(B) $F(0)$
(C) $F(a)$
(D) $F'(0)$
(E) $F'(a)$



7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

8. $\displaystyle\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\dfrac{3}{2}$

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{3}{2}$

(E) divergent

9. Which of the following series converge to 2?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

11. Which of the following integrals gives the length of the graph $y = \sin\left(\sqrt{x}\right)$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\int_a^b \sqrt{x + \cos^2\left(\sqrt{x}\right)}\, dx$

(B) $\int_a^b \sqrt{1 + \cos^2\left(\sqrt{x}\right)}\, dx$

(C) $\int_a^b \sqrt{\sin^2\left(\sqrt{x}\right) + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(E) $\int_a^b \sqrt{\frac{1 + \cos^2\left(\sqrt{x}\right)}{4x}}\, dx$

12. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)\, d\theta$

(C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(D) $\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(E) $\int_{7\pi/6}^{-\pi/6} (1 + 2\sin\theta)\, d\theta$

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \dfrac{x^2}{2} - \dfrac{x^3}{3}$

(B) $1 - x + \dfrac{x^2}{2}$

(C) $x - \dfrac{x^2}{2} + \dfrac{x^3}{3}$

(D) $-1 + x - \dfrac{x^2}{2}$

(E) $-x + \dfrac{x^2}{2} - \dfrac{x^3}{3}$

14. If $\dfrac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. Following are the directions for Section I, Part B, and a representative set of 10 questions.

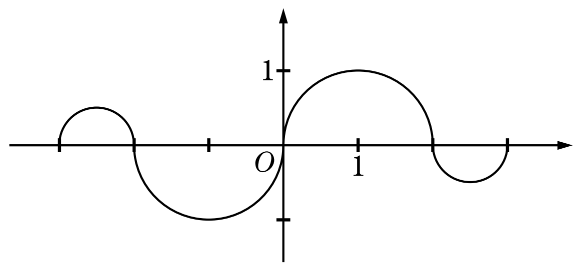
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

15. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\,dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

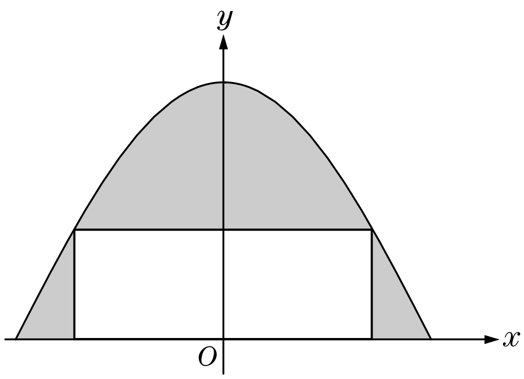
(A) f is continuous at $x = a$.

(B) $\lim_{x \to a} f(x)$ exists.

(C) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists.

(D) $f'(a)$ is defined.

(E) $f''(a)$ is defined.



17. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

    (A) 0.799
    (B) 0.878
    (C) 1.140
    (D) 1.439
    (E) 2.000

18. A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

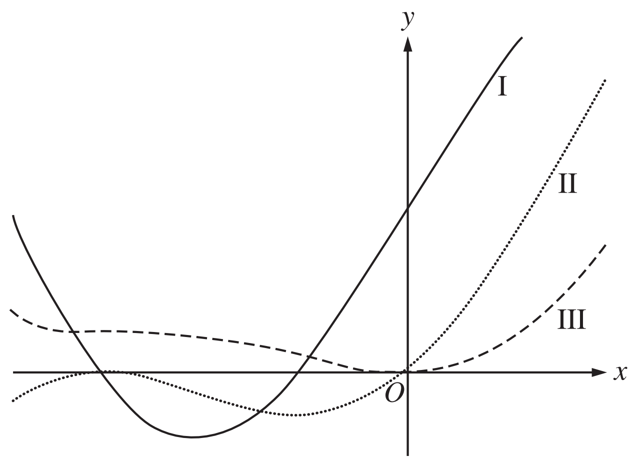
    (A) $\displaystyle\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

    (B) $\displaystyle 2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

    (C) $\displaystyle 2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

    (D) $\displaystyle\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

    (E) $\displaystyle\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$



19. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) -3.268
(B) -1.585
(C) 1.732
(D) 6.585
(E) 11.585

22. Let g be the function given by $g(x) = \int_1^x 100\left(t^2 - 3t + 2\right)e^{-t^2}\,dt$.

Which of the following statements about g must be true?

I. g is increasing on (1, 2).

II. g is increasing on (2, 3).

III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. For a series S, let

$$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$$

$$\text{where } a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements are true?

I. S converges because the terms of S alternate and $\lim_{n\to\infty} a_n = 0$.

II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.

III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III only

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.
For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A		*Part B*	
†1.	D	15.	A
2.	B	16.	E
†3.	D	17.*	B
†4.	D	18.	D
5.	D	19.	E
6.	E	20.*	C
7.	E	21.*	B
†8.	E	22.*	B
†9.	E	†23.	D
10.	E	24.*	B
†11.	D		
†12.	A		
†13.	A		
14.	C		

*Indicates a graphing calculator-active question.
†Indicates a Calculus BC–only topic.

Calculus AB and Calculus BC: Section II

The instructions below are from the 2012 exams. The free-response problems are from the 2008 exams and include information on scoring. The 2008 exam format was three questions in Part A (45 minutes) and three questions in Part B (45 minutes). Additional sample questions can be found at AP Central.

Instructions for Section II

Total Time	**1 hour, 30 minutes**
Number of Questions	6
Percent of Total Score	50%
Writing Instrument	Either pencil or pen with black or dark blue ink
Weight	The questions are weighted equally, but the parts of a question are not necessarily given equal weight.
	Part A
Number of Questions	2
Time	30 minutes
Electronic Devices	Graphing calculator required
	Part B
Number of Questions	4
Time	60 minutes
Electronic Devices	None allowed

The questions for Section II are printed in the booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely, the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

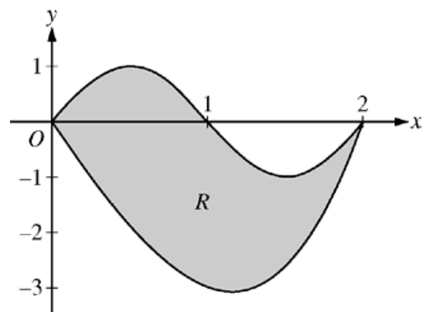
- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For more information on the instructions for the free-response sections, read the "2011 FRQ Instruction Commentary," which is available on the Calculus AB and Calculus BC Exam Pages at AP Central.

Calculus AB Sample Free-Response Questions

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour	$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$
(b) The average number of people waiting in line during the first 4 hours is approximately $\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$ $= 155.25$ people	$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$
(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.	$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$
OR	OR
The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.	$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$
[*Note: There is a function* L *that satisfies the given conditions with* $L'(t) = 0$ *for exactly three values of* t.]	
(d) $\int_0^3 r(t)\,dt = 972.784$ There were approximately 973 tickets sold by 3 P.M.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Question 3

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

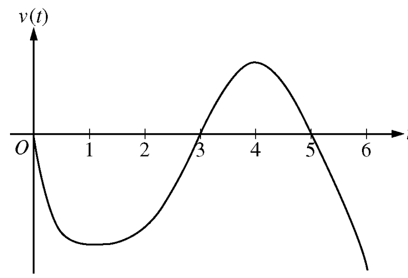
(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000 \text{ cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5 \text{ cm}/\text{min}$. $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$ $2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$ $\frac{dh}{dt} = 0.038 \text{ or } 0.039 \text{ cm}/\text{min}$	$4: \begin{cases} 1: \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2: \text{expression for } \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$. This occurs when $t = 25$ minutes. Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.	$3: \begin{cases} 1: R(t) = 2000 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\, dt$.	$2: \begin{cases} 1: \text{limits and initial condition} \\ 1: \text{integrand} \end{cases}$

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

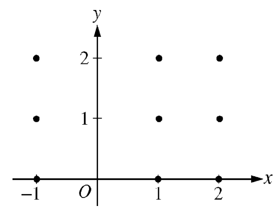
(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$ $x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	$3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

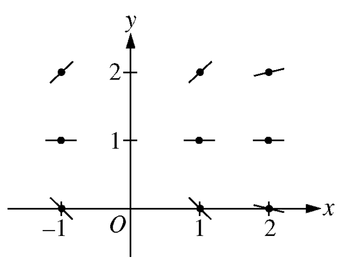
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^C e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^C$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)},\ x > 0$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

(c) $\lim_{x\to\infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

Question 6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

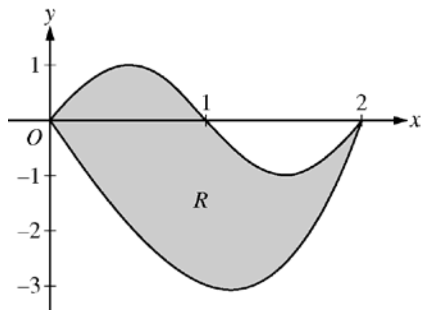
(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

(a) $f\left(e^2\right) = \dfrac{\ln e^2}{e^2} = \dfrac{2}{e^2}$, $f'\left(e^2\right) = \dfrac{1 - \ln e^2}{\left(e^2\right)^2} = -\dfrac{1}{e^4}$ An equation for the tangent line is $y = \dfrac{2}{e^2} - \dfrac{1}{e^4}\left(x - e^2\right)$.	$2 : \begin{cases} 1 : f\left(e^2\right) \text{ and } f'\left(e^2\right) \\ 1 : \text{answer} \end{cases}$
(b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.	$3 : \begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$
(c) $f''(x) = \dfrac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \dfrac{-3 + 2\ln x}{x^3}$ for all $x > 0$ $f''(x) = 0$ when $-3 + 2\ln x = 0$ $x = e^{3/2}$ The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.	$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$
(d) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = -\infty$ or Does Not Exist	1 : answer

Calculus BC Sample Free-Response Questions

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour	$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$
(b) The average number of people waiting in line during the first 4 hours is approximately $\frac{1}{4}\left(\frac{L(0)+L(1)}{2}(1-0) + \frac{L(1)+L(3)}{2}(3-1) + \frac{L(3)+L(4)}{2}(4-3)\right)$ $= 155.25$ people	$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$
(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.	$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$
OR	OR
The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.	$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$
[*Note: There is a function* L *that satisfies the given conditions with* $L'(t) = 0$ *for exactly three values of* t.]	
(d) $\int_0^3 r(t)\,dt = 972.784$ There were approximately 973 tickets sold by 3 P.M.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Question 3

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

(a) $P_1(x) = 80 + 128(x - 2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \le x \le 3$.

$$4 : \begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$$

(b) $P_3(x) = 80 + 128(x - 2) + \frac{488}{6}(x - 2)^2 + \frac{448}{18}(x - 2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

$$3 : \begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$$

(c) The fourth derivative of h is increasing on the interval $1 \le x \le 3$, so $\max\limits_{1.9 \le x \le 2} \left| h^{(4)}(x) \right| = \frac{584}{9}$.

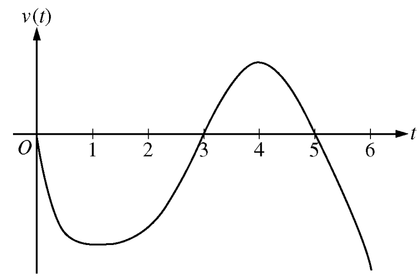
Therefore, $|h(1.9) - P_3(1.9)| \le \frac{584}{9} \frac{|1.9 - 2|^4}{4!}$

$= 2.7037 \times 10^{-4}$

$< 3 \times 10^{-4}$

$$2 : \begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$$

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$ $x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	$3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

Question 5

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

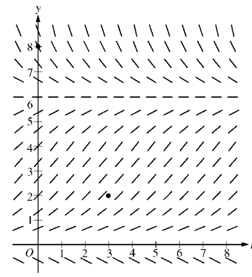
(c) Find the value of $f(3)$.

(a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$ Therefore, f has a relative minimum at $x = 3$.	$2 : \begin{cases} 1\text{: minimum at } x = 3 \\ 1\text{: justification} \end{cases}$
(b) $f''(x) = e^x + (x-3)e^x = (x-2)e^x$ $f''(x) > 0$ for $x > 2$ $f'(x) < 0$ for $0 < x < 3$ Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.	$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$
(c) $f(3) = f(1) + \int_1^3 f'(x)\,dx = 7 + \int_1^3 (x-3)e^x\,dx$ $u = x-3 \quad dv = e^x\,dx$ $du = dx \quad v = e^x$ $f(3) = 7 + (x-3)e^x\Big\vert_1^3 - \int_1^3 e^x\,dx$ $= 7 + \left((x-3)e^x - e^x\right)\Big\vert_1^3$ $= 7 + 3e - e^3$	$4 : \begin{cases} 1 : \text{uses initial condition} \\ 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$

Question 6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)

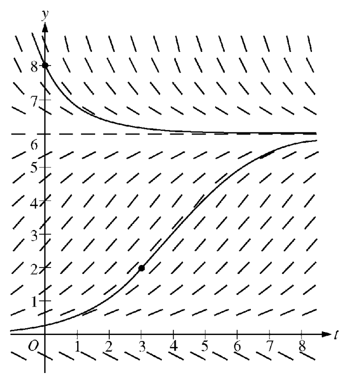


(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

(a)



$2 : \begin{cases} 1\text{: solution curve through } (0,8) \\ 1\text{: solution curve through } (3,2) \end{cases}$

(b) $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation of } f(1) \end{cases}$

(c) $\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6 - y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$

$f(0) = 8;\ f'(0) = \left.\frac{dy}{dt}\right|_{t=0} = \frac{8}{8}(6 - 8) = -2;$ and

$f''(0) = \left.\frac{d^2y}{dt^2}\right|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$

The second-degree Taylor polynomial for f about $t = 0$ is $P_2(t) = 8 - 2t + \frac{5}{4}t^2$.

$f(1) \approx P_2(1) = \frac{29}{4}$

$4 : \begin{cases} 2 : \frac{d^2y}{dt^2} \\ 1 : \text{second-degree Taylor polynomial} \\ 1 : \text{approximation of } f(1) \end{cases}$

(d) The range of f for $t \geq 0$ is $6 < y \leq 8$.

1 : answer

Teacher Support

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- AP Course Descriptions, information about the AP Course Audit, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.
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- The Course Home Pages (apcentral.collegeboard.org/coursehomepages), which contain articles, teaching tips, activities, lab ideas, and other course-specific content contributed by colleagues in the AP community.
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Additional Resources

Teacher's Guides and **Course Descriptions** may be downloaded free of charge from AP Central; printed copies may be purchased through the College Board Store (store.collegeboard.org).

Course Audit Resources. For those looking for information on developing syllabi, the AP Course Audit website offers a host of valuable resources. Each subject has a syllabus development guide that includes the guidelines reviewers use to evaluate syllabi as well as multiple samples of evidence for each requirement. Four sample syllabi written by AP teachers and college faculty who teach the equivalent course at colleges and universities are also available. Along with a syllabus self-evaluation checklist and an example textbook list, a set of curricular/resource requirements is provided for each course that outlines the expectations that college faculty nationwide have established for college-level courses. Visit www.collegeboard.org/apcourseaudit for more information and to download these free resources.

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